

Unruh effect for neutrinos interacting with accelerated matter

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ABSTRACT: We study the evolution of neutrinos in a background matter moving with a linear acceleration. The Dirac equation for a massive neutrino electroweakly interacting with background fermions is obtained in a comoving frame where matter is at rest. We solve this Dirac equation for ultrarelativistic neutrinos. The neutrino quantum states in matter moving with a linear acceleration are obtained. We demonstrate that the neutrino electroweak interaction with an accelerated matter leads to the vacuum instability which results in the neutrino-antineutrino pairs creation. We rederive the temperature of the Unruh radiation and find the correction to the Unruh effect due to the specific neutrino interaction with background fermions. As a possible application of the obtained results we discuss the neutrino pairs creation in a core collapsing supernova. The astrophysical upper limit on the neutrino masses is obtained.

KEYWORDS: Neutrino Physics, Integrable Equations in Physics, Classical Theories of Gravity

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1 Introduction

The standard model neutrino interaction with other fermions is known to be extremely weak. Nevertheless, in some cases, even this weak interaction is crucial in the evolution of a neutrino system. One can recall the resonant amplification of neutrino flavor oscillations in background matter – the Mikheyev-Smirnov-Wolfenstein (MSW) effect [1, 2] – which is the most plausible explanation of the solar neutrinos deficit [3]. Typically background matter, which a neutrino interacts with, is considered to either be at rest or move with a constant velocity. In both cases matter effects can influence the neutrino oscillations picture.

Recently neutrino flavor oscillations in matter moving with an acceleration was discussed in refs. [4–6]. The modification of the neutrino refraction index owing to a nonzero matter acceleration was accounted for in refs. [4, 5] without consideration of possible noninertial effects. The neutrino motion in an accelerated frame accounting for noninertial effects was described in ref. [6]. The influence of the Unruh radiation on neutrino flavor oscillations was discussed in ref. [7].

Linearly accelerated particles reveal another interesting effect consisting in the emission of a thermal radiation, which was first predicted by Unruh [8]. The temperature of this radiation appears to be proportional to the particle acceleration. This effect is still widely discussed in the literature (for a recent review see, e.g., ref. [9]). In ref. [10], it was shown that the Unruh effect strongly depends on the way to observe a thermal radiation. Nevertheless there are some suggestions how to detect the Unruh radiation experimentally [11].

In the present work we continue to study the influence of noninertial effects on electroweakly interacting particles. In refs. [6, 12] we considered the electroweak interaction of neutrinos, electrons, and quarks with a rotating background matter in a corotating frame. In particular, the new galvano-rotational effect was predicted in ref. [12]. In this work we study the neutrino interaction with a linearly accelerated matter. Note that this kind of the matter acceleration can be implemented in various astrophysical media such as a supernova (SN) at the bounce stage [13] and jets from active galactic nuclei [14]. One can also expect intense neutrino beams in these environments.

This work is organized as follows. In section 2 we recall the standard model neutrino interaction with background fermions which can move with a constant velocity. In section 3 we derive and solve the Dirac equation, accounting for noninertial effects, for ultrarelativistic neutrinos interacting with a linearly accelerated matter. In section 4 we study the neutrino quantum states in a linearly accelerated matter. The neutrino-antineutrino ($\nu\bar{\nu}$) pairs creation is described in section 5. In particular, we reproduce the Unruh effect for neutrinos in electroweakly interacting matter moving with a linear acceleration and consider the correction to this effect owing to the specific neutrino interaction. In section 6 we propose the astrophysical application of our results, consisting in the $\nu\bar{\nu}$ pairs creation in a SN explosion at the bounce stage. Finally, in section 7 we discuss our results. Some useful mathematical formulas are provided in appendix A.

2 Neutrino interaction with matter in an inertial frame

In this section we shall briefly remind how active neutrinos interact with background matter in the flat space-time. We describe the neutrino electroweak interaction in the Fermi approximation.

The electroweak interaction of the flavor neutrino eigenstates ν_α , $\alpha = e, \mu, \tau$, with a background matter consisting of electrons e , protons p , and neutrons n , can be described in the mean field approximation by the Langrangian

$$\mathcal{L}_{\text{int}} = - \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma_\mu^L \nu_\alpha \cdot J_{\nu_\alpha}^\mu, \quad (2.1)$$

where $\gamma_\mu^L = \gamma_\mu (1 - \gamma^5)/2$, $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$ are the Dirac matrices, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The effective current in eq. (2.1) has the form [15],

$$J_{\nu_\alpha}^\mu = \sqrt{2}G_F \sum_{f=e,p,n} \left(q_f^{(1)} j_f^\mu + q_f^{(2)} \lambda_f^\mu \right), \quad (2.2)$$

where $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant, j_f^μ is the hydrodynamic current of background fermions, λ_f^μ is the four vector of the mean polarization. The hydrodynamic current $j_f^\mu = n_f u_f^\mu$ depends on the invariant number density n_f , i.e. the density in the rest frame of background fermions, and the macroscopic mean four velocity u_f^μ of these fermions. The explicit form of λ_f^μ in terms of the invariant density, four velocity, and the invariant polarization ζ_f can be found, e.g., in ref. [16]. The coefficients $q_f^{(1,2)}$ in eq. (2.2)

for ν_e have the form [15]

$$q_1^{(f)} = I_{L3}^{(f)} - 2Q_f\xi + \delta_{ef}, \quad q_2^{(f)} = -I_{L3}^{(f)} - \delta_{ef}, \quad (2.3)$$

where $I_{L3}^{(f)}$ is the third component of the weak isospin of type f fermions, Q_f is the value of their electric charge, $\xi = \sin^2 \theta_W \approx 0.23$ is the Weinberg parameter, $\delta_{ef} = 1$ for electrons and vanishes for protons and neutrons. To get $q_f^{(1,2)}$ for $\nu_{\mu,\tau}$ interactions with the same background fermions, we should set $\delta_{ef} = 0$ in eq. (2.3).

The Lagrangian in eq. (2.1) can be obtained by the averaging of the fermionic vector and axial-vector currents in the rest frame of background fermions. Therefore, fermions, contributing to eq. (2.2), are supposed to have constant and homogeneous mean velocities. Otherwise (e.g, for matter moving with an acceleration; cf. section 3 below), eq. (2.2) is not valid. Indeed to obtain eq. (2.2) one should make a Lorentz boost to the rest frame of background fermions to make the averaging and then make a boost back to the laboratory frame. Such coordinate transformations are undefined if the Lorentz symmetry is broken.

In our work we shall be mainly interested in the neutrino evolution in the dense electrically neutral nuclear matter of a protoneutron star (PNS). Let us assume that this matter is at rest and unpolarized. In this case, on the basis of eqs. (2.2) and (2.3) we get the effective potential $V_{\nu_\alpha} = J_{\nu_\alpha}^0$ as

$$V_{\nu_\alpha} \approx -\frac{G_F}{\sqrt{2}} n_n, \quad (2.4)$$

where n_n is the neutron density. Note that the effective potentials for different neutrino types are approximately equal since the densities of charged particles inside PNS are much lower than that of neutrons.

In the wake of the results of the recent experiments (see, e.g., ref. [17]), it is commonly believed that massive neutrino eigenstates, i.e. those which have definite masses, are the mixture of flavor neutrinos ν_α defined above. It should be noted that the matter interaction of mass eigenstates is nondiagonal in the particle species. In the present work we shall not distinguish between mass and flavor eigenstates. For instance, this approximation is valid if we study the evolution of the $\nu_e - \nu_\tau$ system, where the mixing angle θ_{13} is known to be the smallest. Alternatively we may consider the neutrino system with the parameters far away from any MSW resonance.

According to many theoretical models for the neutrino mass generation [18], the neutrino mass eigenstates are likely to be Majorana particles. Nevertheless, in the present work, we shall assume that the neutrino mass eigenstates ψ_i , $i = 1, 2, \dots$, are the Dirac particles. Note that, despite the great experimental efforts [19], the issue whether neutrinos are Dirac or Majorana particles still remains open.

3 Dirac equation for a neutrino interacting with linearly accelerated matter

In this section we shall describe the motion of a neutrino in a linearly accelerated matter. Our analysis will be based on the exact solution of the Dirac equation accounting for noninertial effects.

The neutrino interaction with background matter described in section 2 implied the uniform matter motion with a constant velocity, i.e. $J_{\nu_\alpha}^\mu$ in eq. (2.2) depends on neither coordinates nor time. The opposite situation would mean an accelerated matter. As was mentioned in section 2, for accelerated matter we cannot assume a nonzero $\mathbf{J}_{\nu_\alpha}(\mathbf{r}, t)$ in eq. (2.1) and in the corresponding Dirac equation, written in the flat space-time, since the Lorentz symmetry is broken.

Nevertheless, even for accelerated matter, we can always find a reference frame where matter is at rest. Assuming that matter is unpolarized, we get that only $J_{\nu_\alpha}^0 \neq 0$ exists in this reference frame. Thus, in this frame, we can still use the effective potential defined in eq. (2.4) since it depends on the invariant density of background fermions. However, in this case the Dirac equation should be modified to account for noninertial effects. This approach was developed in refs. [6, 12], where we studied the propagation of electroweakly interacting particles (including neutrinos) in a rotating matter.

It is known that the description of the particle motion by an accelerated observer is equivalent to the interaction of this particle with an effective gravitational field. Hence, in our treatment of the neutrino evolution, we should rewrite the Dirac equation in the curved space-time corresponding to a linearly accelerated frame, i.e in the Rindler space-time. If we suppose that the matter is accelerated along the z -axis, the interval in the Rindler space-time has the form [20],

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2 z^2 dt^2 - dx^2 - dy^2 - dz^2, \quad (3.1)$$

where $g_{\mu\nu}$ is the metric tensor of the effective gravitational field and a is the proper acceleration of matter. Here we use the Cartesian coordinates $x^\mu = (t, x, y, z)$. The interval in eq. (3.1) can be transformed to the Minkowskian one $ds^2 = d\mathcal{T}^2 - d\mathcal{X}^2 - d\mathcal{Y}^2 - d\mathcal{Z}^2$ by changing the coordinates

$$\begin{aligned} \mathcal{T} &= z \sinh at, & \mathcal{Z} &= z \cosh at, & \mathcal{X} &= x, & \mathcal{Y} &= y. \\ t &= \frac{1}{a} \operatorname{arctanh} \frac{\mathcal{T}}{\mathcal{Z}}, & z &= \sqrt{\mathcal{Z}^2 - \mathcal{T}^2}, & x &= \mathcal{X}, & y &= \mathcal{Y}. \end{aligned} \quad (3.2)$$

It should be noted that the metric in eq. (3.1) does not span over the all space-time. It covers only the sector $\mathcal{Z} > |\mathcal{T}|$ in the $(\mathcal{Z}, \mathcal{T})$ hyperplane.

The Dirac equation for a neutrino interacting with background matter in a curved space-time can be obtained, using the results of refs. [6, 12, 21] and accounting for eq. (2.1), in the form,

$$[i\gamma^\mu(x)\nabla_\mu - m]\psi = \frac{1}{2}J^\mu\gamma_\mu(x)[1 - \gamma^5(x)]\psi, \quad (3.3)$$

where ψ is the neutrino bispinor, m is the neutrino mass, $\gamma^\mu(x)$ are the coordinate dependent Dirac matrices, $\nabla_\mu = \partial_\mu + \Gamma_\mu$ is the covariant derivative, Γ_μ is the spin connection, $\gamma^5(x) = -(i/4!)E^{\mu\nu\alpha\beta}\gamma_\mu(x)\gamma_\nu(x)\gamma_\alpha(x)\gamma_\beta(x)$, $E^{\mu\nu\alpha\beta} = \varepsilon^{\mu\nu\alpha\beta}/\sqrt{-g}$ is the covariant anti-symmetric tensor in curved space-time, $g = \det(g_{\mu\nu})$, and J^μ is the effective external current of background fermions; cf. eq. (2.2). Note that, since we choose a comoving frame, only the zeroth component of J^μ is nonvanishing: $J^0 = V \neq 0$, where V is given in eq. (2.4). In eq. (3.3) we omit the index ν_α for brevity: $V \equiv V_{\nu_\alpha}$, $m \equiv m_{\nu_\alpha}$ etc.

One can check that the metric tensor in eq. (3.1) can be diagonalized using the following vierbein vectors:

$$e_0^\mu = \left(\frac{1}{az}, 0, 0, 0 \right), \quad e_1^\mu = (0, 1, 0, 0), \quad e_2^\mu = (0, 0, 1, 0), \quad e_3^\mu = (0, 0, 0, 1). \quad (3.4)$$

Indeed, the direct calculation using eq.(3.4) shows that $\eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu}$, where $\eta_{ab} = \text{diag}(1, -1, -1, -1)$ is the metric in a locally Minkowskian frame.

To obtain the spin connection one should introduce the constant Dirac matrices in a locally Minkowskian frame as $\gamma^{\bar{a}} = e_{\bar{\mu}}^a \gamma^\mu(x)$. As shown in ref. [6], $\gamma^5(x) = i\gamma^{\bar{0}}\gamma^{\bar{1}}\gamma^{\bar{2}}\gamma^{\bar{3}} = \gamma^{\bar{5}}$ does not depend on coordinates. The spin connection in the Dirac eq. (3.3) has the form [21],

$$\Gamma_\mu = -\frac{i}{4}\sigma^{ab}\omega_{ab\mu}, \quad \omega_{ab\mu} = e_a^\nu e_{b\nu;\mu}, \quad (3.5)$$

where $\sigma_{ab} = (i/2)[\gamma_{\bar{a}}, \gamma_{\bar{b}}]_-$ are the generators of the Lorentz transformations in a locally Minkowskian frame and the semicolon stays for the covariant derivative. The explicit calculation on the basis of eq. (3.5) shows that the nonzero components of the connection one-form $\omega_{ab} = \omega_{ab\mu}dx^\mu$ are

$$\omega_{01\mu} = -\omega_{10\mu} = (a, 0, 0, 0). \quad (3.6)$$

Using eqs. (3.5) and (3.6) we get that $i\gamma^\mu(x)\Gamma_\mu = i\gamma^{\bar{3}}/2z$.

Using the definition of $\gamma^{\bar{a}}$, the Dirac eq. (3.3) takes the form,

$$\left[i\gamma^{\bar{0}}\frac{\partial_0}{az} + i\gamma^{\bar{1}}\partial_x + i\gamma^{\bar{2}}\partial_y + i\gamma^{\bar{3}}\left(\partial_z + \frac{1}{2z}\right) - m \right] \psi = \frac{1}{2}az\gamma^{\bar{0}}V(1 - \gamma^{\bar{5}})\psi. \quad (3.7)$$

Analogous Dirac equation in the Rindler wedge with $V = 0$ was studied in ref. [22]. Since eq. (3.7) does not explicitly depend on t, y , and z , we shall look for its solution in the form,

$$\psi = \exp(-iEt + ip_x x + ip_y y) \psi_z, \quad (3.8)$$

where $\psi_z = \psi_z(z)$ is the wave function depending on z .

It is convenient to rewrite eq. (3.7) as

$$[\gamma^{\bar{a}}Q_a - m + U] \psi_z = 0, \quad (3.9)$$

where $Q^a = q^a - q_{\text{eff}}A_{\text{eff}}^a$, q_{eff} is the effective electric charge, $q^a = (0, p_x, p_y, -i\partial_z)$,

$$A_{\text{eff}}^a = \frac{1}{q_{\text{eff}}} \left(\frac{azV}{2} - \frac{E}{az}, 0, 0, \frac{i}{2z} \right), \quad (3.10)$$

is the potential of the effective electromagnetic field, and $U = azV\gamma^{\bar{0}}\gamma^{\bar{5}}/2$.

Let us look for the solution of eq. (3.9) in the form, $\psi_z = [\gamma^{\bar{a}}Q_a + m - U]\Phi$, where Φ is the new spinor. The equation for Φ reads

$$\left[\left(\partial_z + \frac{1}{2z} \right)^2 + \left(\frac{E}{az} - \frac{azV}{2} \right)^2 - p_\perp^2 + \frac{a^2 z^2 V^2}{4} - m^2 \right. \\ \left. + \left(\frac{E}{az^2} + \frac{aV}{2} \right) i\alpha_3 - \frac{aV}{2} \left[2z \left(\frac{E}{az} - \frac{azV}{2} \right) - i\alpha_3 \right] \gamma^{\bar{5}} + mazV\gamma^{\bar{0}}\gamma^{\bar{5}} \right] \Phi = 0, \quad (3.11)$$

where $p_\perp^2 = p_x^2 + p_y^2$, $\alpha_3 = \gamma^{\bar{5}}\Sigma_3$, and $\Sigma_3 = \gamma^{\bar{0}}\gamma^{\bar{3}}\gamma^{\bar{5}}$.

The solution of eq. (3.11) can be found for ultrarelativistic particles. In the limit $m \rightarrow 0$, we can represent $\Phi = v\varphi$ in eq. (3.11), where $\varphi = \varphi(z)$ is a scalar function and v is a constant spinor satisfying $\Sigma_3 v = \sigma v$ and $\gamma^{\bar{5}} v = \chi v$, with $\sigma = \pm 1$ and $\chi = \pm 1$, since both Σ_3 and $\gamma^{\bar{5}}$ now commute with the operator of eq. (3.11).

We shall be interested in the description of left active neutrinos satisfying $(1 + \gamma^{\bar{5}})\psi = 0$ and having $\chi = +1$. One can show that, for right sterile neutrinos with $\chi = -1$, the interaction with matter is washed out. Using the new variable $\rho = |V|az^2$ we can write the equation for φ_σ as

$$\left[\rho \partial_\rho^2 + \partial_\rho - \frac{\mu^2}{\rho} + \frac{\rho}{4} - \kappa \right] \varphi_\sigma = 0, \quad (3.12)$$

where

$$\kappa = \kappa_0 + s\kappa_1 - i\frac{\sigma s}{4}, \quad \kappa_0 = \frac{p_\perp^2 + m^2}{4|V|a}, \quad \kappa_1 = \frac{E}{2a}, \quad \mu = \frac{1}{4} - i\sigma\kappa_1, \quad (3.13)$$

and $s = \text{sgn}(V) = -1$ for neutrinos in a neutron matter; cf. eq. (2.4).

4 Classification of the neutrino quantum states

In this section we shall find the solution of the wave equation for neutrinos interacting with accelerated matter in the explicit form and classify the neutrino quantum states.

The solutions of eq. (3.12) have the form,

$$\zeta\varphi_\sigma = \frac{1}{\sqrt{\rho}} M_{i\zeta\kappa, \mu}(\zeta i\rho), \quad \zeta\varphi_\sigma = \frac{1}{\sqrt{\rho}} W_{i\zeta\kappa, \mu}(i\zeta\rho), \quad \zeta = \pm, \quad (4.1)$$

where $M_{\lambda, \mu}(z)$ and $W_{\lambda, \mu}(z)$ are the Whittaker functions. Note that solutions of the Dirac equation, expressed via Whittaker functions, in the $1 + 1$ de Sitter space-time were also obtained in ref. [23].

We shall suppose that the matter acceleration takes place in the spatial interval $z_{\text{in}} < z < z_{\text{out}}$, where $z_{\text{in}} \ll z_{\text{out}}$. It corresponds to the change of the dimensionless variable $\rho_{\text{in}} < \rho < \rho_{\text{out}}$, where $\rho_{\text{in}} \ll 1$ and $\rho_{\text{out}} \gg 1$. Using eq. (A.1), we get that $\zeta\varphi$ can be identified with the quantum state corresponding to a definite sign of the momentum projection along the z -axis at $\rho \sim \rho_{\text{in}}$, i.e. an in-state, whereas $\zeta\varphi$ is the wave function of this quantum state at $\rho \sim \rho_{\text{out}}$. We shall call it an out-state.

Therefore we can define two sets of the orthogonal neutrino wave functions: $\pm\psi_n$ and $\pm\psi_n$ (in and out solutions). These wave functions can be obtained from $\pm\varphi$ and $\pm\varphi$ using eq. (3.8) and the relation between ψ_z and Φ in section 3. We shall denote all the quantum numbers, like $p_{x,y}$, E , and σ , as n for brevity.

The secondly quantized neutrino field $\hat{\psi}$ can be decomposed using $\pm\psi_n$ and $\pm\psi_n$ in the form,

$$\hat{\psi} = \sum_n \left[\hat{a}_n(\text{in})_+ \psi_n + \hat{b}_n^\dagger(\text{in})_- \psi_n \right] = \sum_n \left[\hat{a}_n(\text{out})^+ \psi_n + \hat{b}_n^\dagger(\text{out})^- \psi_n \right], \quad (4.2)$$

where \hat{a}_n and \hat{b}_n are the annihilation operators of the quantum state corresponding to a definite sign of the momentum projection along the z -axis. These operators act on quantum

states in “in” and “out” Fock spaces. The vacuum in these Fock spaces is defined as

$$\hat{a}_n(\text{in, out})|0\rangle_{\text{in, out}} = \hat{b}_n(\text{in, out})|0\rangle_{\text{in, out}} = 0. \quad (4.3)$$

The creation and annihilation operators obey the usual anticommutation relations,

$$\left[\hat{a}_n(\text{in, out}), \hat{a}_{n'}^\dagger(\text{in, out}) \right]_+ = \left[\hat{b}_n(\text{in, out}), \hat{b}_{n'}^\dagger(\text{in, out}) \right]_+ = \delta_{n, n'}, \quad (4.4)$$

with all the other anticommutators being equal to zero.

Now, let us find ${}^\pm\psi_n$ and ${}^\pm\psi_n$ in the explicit form. For this purpose we represent the bispinor ψ_z of active ultrarelativistic neutrinos as $\psi_z^T = (0, \eta)$. Here we assume the Dirac matrices in the chiral representation [24]

$$\gamma^{\bar{0}} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^{\bar{k}} = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^{\bar{5}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (4.5)$$

where $\sigma_k = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. The two component spinors η_σ corresponding to the opposite helicity states with $\sigma = \pm 1$, can be obtained if we choose the following constant bispinors v_σ :

$$v_+^T = (1, 0, 0, 0), \quad v_-^T = (0, 1, 0, 0). \quad (4.6)$$

Finally, using eqs. (4.5) and (4.6), we get η_\pm in the form,

$$\eta_+ = \Pi \begin{pmatrix} \varphi_+ \\ 0 \end{pmatrix}, \quad \eta_- = \Pi \begin{pmatrix} 0 \\ \varphi_- \end{pmatrix}, \quad (4.7)$$

where

$$\Pi = 2\sqrt{a|V|} \left[s \frac{\sqrt{\rho}}{2} - \frac{E}{2a\sqrt{\rho}} + i\sigma \left(\sqrt{\rho}\partial_\rho + \frac{1}{4\sqrt{\rho}} \right) \right] - (\boldsymbol{\sigma}_\perp \mathbf{p}_\perp), \quad (4.8)$$

is the projection operator, $\boldsymbol{\sigma}_\perp = (\sigma_1, \sigma_2)$, and $\mathbf{p}_\perp = (p_x, p_y)$.

Let us first obtain out-states in the explicit form. Using eqs. (4.1), (4.7), and (4.8), as well as eq. (A.3), we can obtain the wave functions corresponding to the negative momentum along the z -axis,

$$\begin{aligned} {}^-\eta_+ &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} {}^-\alpha_+ W_{-1/4+i\kappa_0-i\kappa_1, 1/4-i\kappa_1}(i\rho) + {}^-\beta_+ W_{-3/4+i\kappa_0-i\kappa_1, -1/4-i\kappa_1}(i\rho) \\ -(p_x + ip_y) W_{-1/4+i\kappa_0-i\kappa_1, 1/4-i\kappa_1}(i\rho) \end{pmatrix}, \\ {}^-\eta_- &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} -(p_x - ip_y) W_{1/4+i\kappa_0-i\kappa_1, 1/4+i\kappa_1}(i\rho) \\ {}^-\alpha_- W_{1/4+i\kappa_0-i\kappa_1, 1/4+i\kappa_1}(i\rho) + {}^-\beta_- W_{3/4+i\kappa_0-i\kappa_1, -1/4+i\kappa_1}(i\rho) \end{pmatrix}. \end{aligned} \quad (4.9)$$

Analogously one can derive the wave functions for the positive projection of the momentum as

$$\begin{aligned} {}^+\eta_+ &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} {}^+\alpha_+ W_{1/4-i\kappa_0+i\kappa_1, 1/4-i\kappa_1}(-i\rho) + {}^+\beta_+ W_{3/4-i\kappa_0+i\kappa_1, -1/4-i\kappa_1}(-i\rho) \\ -(p_x + ip_y) W_{1/4-i\kappa_0+i\kappa_1, 1/4-i\kappa_1}(-i\rho) \end{pmatrix}, \\ {}^+\eta_- &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} -(p_x - ip_y) W_{-1/4-i\kappa_0+i\kappa_1, 1/4+i\kappa_1}(-i\rho) \\ {}^+\alpha_- W_{-1/4-i\kappa_0+i\kappa_1, 1/4+i\kappa_1}(-i\rho) + {}^+\beta_- W_{-3/4-i\kappa_0+i\kappa_1, -1/4+i\kappa_1}(-i\rho) \end{pmatrix}. \end{aligned} \quad (4.10)$$

The coefficients in eqs. (4.15) and (4.10) have the form,

$$\begin{aligned}\zeta_{\alpha_{\zeta'}} &= -2E\sqrt{\frac{|V|}{a\rho}}, \quad \pm\beta_{\pm} = 2\sqrt{a|V|}e^{\mp 3\pi i/4}, \\ \pm\beta_{\mp} &= 2\sqrt{a|V|}e^{\pm 3\pi i/4} \left(\kappa_0 - 2\kappa_1 \mp \frac{i}{2} \right),\end{aligned}\quad (4.11)$$

where ζ and ζ' take the values \pm independently.

We define the norm of the total wave function as

$$\langle \psi | \psi' \rangle = \int d^3x \sqrt{-g} \bar{\psi}_{E,p_x,p_y} \gamma^0(x) \psi_{E',p'_x,p'_y} = \|\psi\|^2 \delta(E - E') \delta^2(\mathbf{p}_{\perp} - \mathbf{p}'_{\perp}), \quad (4.12)$$

where $\bar{\psi} = \psi^\dagger \gamma^0$. The norm of a two component spinor η can be defined analogously. Using eqs. (4.9) and (4.10), we get that

$$\sigma_1 (-\eta_-)^* = {}^+\eta_+, \quad \sigma_1 ({}^+\eta_-)^* = -\eta_+. \quad (4.13)$$

Therefore, one can establish the relation between the norms of the spinors,

$$\|-\eta_-\|^2 = \|{}^+\eta_+\|^2, \quad \|{}^+\eta_-\|^2 = \|-\eta_+\|^2, \quad (4.14)$$

Note that the analog of eqs. (4.13) and (4.14) was found in ref. [25].

The in-states can be obtained in the same manner as eqs. (4.9) and (4.10). We just give the final result,

$$\begin{aligned}-\eta_+ &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} -\alpha_+ M_{-1/4+i\kappa_0-i\kappa_1, 1/4-i\kappa_1}(i\rho) + \beta_+ M_{-3/4+i\kappa_0-i\kappa_1, -1/4-i\kappa_1}(i\rho) \\ -(p_x + ip_y) M_{-1/4+i\kappa_0-i\kappa_1, 1/4-i\kappa_1}(i\rho) \end{pmatrix}, \\ -\eta_- &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} -(p_x - ip_y) M_{1/4+i\kappa_0-i\kappa_1, 1/4+i\kappa_1}(i\rho) \\ -\alpha_- M_{1/4+i\kappa_0-i\kappa_1, 1/4+i\kappa_1}(i\rho) + \beta_- M_{3/4+i\kappa_0-i\kappa_1, -1/4+i\kappa_1}(i\rho) \end{pmatrix},\end{aligned}\quad (4.15)$$

and

$$\begin{aligned}{}^+\eta_+ &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} +\alpha_+ M_{1/4-i\kappa_0+i\kappa_1, 1/4-i\kappa_1}(-i\rho) + \beta_+ M_{3/4-i\kappa_0+i\kappa_1, -1/4-i\kappa_1}(-i\rho) \\ -(p_x + ip_y) M_{1/4-i\kappa_0+i\kappa_1, 1/4-i\kappa_1}(-i\rho) \end{pmatrix}, \\ {}^+\eta_- &= \frac{1}{\sqrt{\rho}} \begin{pmatrix} -(p_x - ip_y) M_{-1/4-i\kappa_0+i\kappa_1, 1/4+i\kappa_1}(-i\rho) \\ +\alpha_- M_{-1/4-i\kappa_0+i\kappa_1, 1/4+i\kappa_1}(-i\rho) + \beta_- M_{-3/4-i\kappa_0+i\kappa_1, -1/4+i\kappa_1}(-i\rho) \end{pmatrix}.\end{aligned}\quad (4.16)$$

The coefficients in eqs. (4.15) and (4.16) are

$$\begin{aligned}\zeta_{\alpha_{\zeta'}} &= -2E\sqrt{\frac{|V|}{a\rho}}, \quad -\beta_{\pm} = e^{\pi i/4} \sqrt{a|V|} (4\kappa_1 \pm i), \\ {}^+\beta_{\pm} &= e^{-\pi i/4} \sqrt{a|V|} (4\kappa_1 \pm i),\end{aligned}\quad (4.17)$$

where ζ and ζ' again take the values \pm independently. Analogously to the out-states, we can establish the following relations between the norms of the wave functions: $\|-\eta_-\|^2 =$

$\|_+\eta_+\|^2$ and $\|_+\eta_-\|^2 = \|_-\eta_+\|^2$. These expressions are valid since it follows from eqs. (4.15) and (4.16) that $\sigma_1(-\eta_-)^* = +\eta_+$ and $\sigma_1(+\eta_-)^* = -\eta_+$.

To complete the determination of the states we need to establish the relation between the norms of the wave functions for the same helicity and different signs of the momentum projection. This relation can be found in the case when $\kappa_1 \gg \kappa_0$. Using eqs. (A.4) and (A.6), we get the ratio of the norms of $^+\eta_-$ and $^-\eta_-$ as

$$\frac{\|_+\eta_-\|^2}{\|_-\eta_-\|^2} = \frac{\pi e^{-2\pi\kappa_1}}{\cosh 2\pi\kappa_1}. \quad (4.18)$$

Note that one can derive the analogous relation between, e.g., the wave functions $^+\eta_-$ and $^-\eta_-$. However, we shall not present it here.

5 Particle creation in an accelerated matter

In this section we describe the creation of $\nu\bar{\nu}$ pairs in a background matter moving with a linear acceleration. We also reproduce the Unruh effect.

Both out- and in-states are the complete sets of orthogonal wave functions. Thus we can expand any in-state wave function over the system of out-states,

$${}_\zeta\eta_\sigma = \sum_{\zeta'=\pm} G\left(\zeta'|\zeta\right) {}_{\zeta'}\eta_\sigma, \quad \zeta = \pm, \quad (5.1)$$

where the Bogoliubov coefficients $G\left(\zeta'|\zeta\right)$ satisfy the unitarity conditions,

$$\sum_{\zeta=\pm} G\left(\zeta'|\zeta\right) G\left(\zeta|\zeta''\right) = \sum_{\zeta=\pm} G\left(\zeta'|\zeta\right) G\left(\zeta|\zeta''\right) = \delta_{\zeta'\zeta''}. \quad (5.2)$$

Here $G\left(\zeta|\zeta'\right) = G\left(\zeta'|\zeta\right)^*$ are the coefficients of the inverse transformation.

The relation between in and out wave functions in eq. (5.1) is equivalent to the Bogoliubov transformation of the creation and annihilation operators defined in eqs. (4.2)-(4.4),

$$\begin{aligned} \hat{a}_n(\text{out}) &= G\left(+|+\right) \hat{a}_n(\text{in}) + G\left(+|-\right) \hat{b}_n^\dagger(\text{in}), \\ \hat{b}_n^\dagger(\text{out}) &= G\left(-|+\right) \hat{a}_n(\text{in}) + G\left(-|-\right) \hat{b}_n^\dagger(\text{in}). \end{aligned} \quad (5.3)$$

Therefore, the nonzero mean value of the number density operator of out-states over the “in” vacuum, ${}_{\text{in}}\langle 0|\hat{a}_n^\dagger(\text{out})\hat{a}_n(\text{out})|0\rangle_{\text{in}} = |G\left(+|-\right)|^2 \neq 0$, can be interpreted as the creation of a $\nu\bar{\nu}$ pair.

From the formal point of view the vacuum instability leading to the $\nu\bar{\nu}$ pairs creation, described in the present work, is analogous to the pairs creation by an external field depending on spatial coordinates. It should be noted that a more detailed analysis of this problem was recently presented in refs. [26, 27].

Setting $\zeta = -$ in eq. (5.1) we get that

$$_-\eta_\sigma = G\left(+|-\right) {}_+\eta_\sigma + G\left(-|-\right) {}_-\eta_\sigma. \quad (5.4)$$

The Bogoliubov coefficients in eq. (5.4) are related to the scalar products, defined in eq. (4.12), of the corresponding in- and out-states as

$$G(+|_-) = \langle +\eta_\sigma | -\eta_\sigma \rangle, \quad G(-|_-) = \langle -\eta_\sigma | -\eta_\sigma \rangle. \quad (5.5)$$

Since we study ultrarelativistic particles, there is a strong correlation between their momentum and spin: neutrinos are left-polarized, whereas antineutrinos are right-polarized. If $\sigma = -1$, the nonzero coefficient $G(+|_-)$ corresponds to the creation of the pair of ν with $(p_z > 0, \sigma = -1)$ and $\bar{\nu}$ with $(p_z < 0, \sigma = -1)$. In case $\sigma = +1$, $G(+|_-)$ describes the creation of ν with $(p_z < 0, \sigma = +1)$ and $\bar{\nu}$ with $(p_z > 0, \sigma = +1)$. However, these two sets of states are identical. Thus, if we define the corresponding probabilities

$$P_\sigma = |G(+|_-)|^2, \quad (5.6)$$

we should get that $P_- + P_+ = 1$. In fact, this relation between the probabilities is the consequence of eq. (4.13).

To derive P_σ in the explicit form, we should use eq. (A.5). For $\sigma = -1$, on the basis of eqs. (4.9), (4.10), and (4.15), we have

$$\begin{aligned} -\eta_- = & \Gamma\left(\frac{3}{2} + 2i\kappa_1\right) \exp\left[\pi(\kappa_0 - \kappa_1) - \frac{i\pi}{4}\right] \\ & \times \left[\frac{\exp(-\pi\kappa_1 + 3\pi i/4)}{\Gamma(1 + i\kappa_0)} -\eta_- + \frac{1}{\Gamma(1/2 + 2i\kappa_1 - i\kappa_0)} +\eta_- \right]. \end{aligned} \quad (5.7)$$

Substituting eq. (5.7) to eq. (5.5) we obtain

$$\frac{G(-|_-)}{G(+|_-)} = \exp\left(-\pi\kappa_1 + \frac{3\pi i}{4}\right) \frac{\|-\eta_-\|^2}{\|+\eta_-\|^2} \frac{\Gamma(1/2 + 2i\kappa_1 - i\kappa_0)}{\Gamma(1 + i\kappa_0)}. \quad (5.8)$$

Analogously, if $\sigma = +1$, one gets that

$$\begin{aligned} -\eta_+ = & \Gamma\left(\frac{3}{2} - 2i\kappa_1\right) \exp\left[\pi(\kappa_0 - \kappa_1) + \frac{i\pi}{4}\right] \\ & \times \left[\frac{\exp(\pi\kappa_1 + 3\pi i/4)}{\Gamma(1/2 - 2i\kappa_1 + i\kappa_0)} -\eta_+ + \frac{1}{\Gamma(1 - i\kappa_0)} +\eta_+ \right], \end{aligned} \quad (5.9)$$

and

$$\frac{G(-|_+)}{G(+|_+)} = \exp\left(\pi\kappa_1 + \frac{3\pi i}{4}\right) \frac{\|-\eta_+\|^2}{\|+\eta_+\|^2} \frac{\Gamma(1 - i\kappa_0)}{\Gamma(1/2 - 2i\kappa_1 + i\kappa_0)}. \quad (5.10)$$

Finally, accounting for eqs. (5.2), (5.6), and (A.6), we get that

$$\begin{aligned} P_- = & \left[1 + e^{-2\pi\kappa_1} \frac{\|-\eta_-\|^4}{\|+\eta_-\|^4} \frac{\sinh \pi\kappa_0}{\kappa_0 \cosh \pi(2\kappa_1 - \kappa_0)} \right]^{-1}, \\ P_+ = & \left[1 + e^{2\pi\kappa_1} \frac{\|-\eta_+\|^4}{\|+\eta_+\|^4} \frac{\kappa_0 \cosh \pi(2\kappa_1 - \kappa_0)}{\sinh \pi\kappa_0} \right]^{-1}. \end{aligned} \quad (5.11)$$

Taking into account eq. (4.14), one can see that $P_- + P_+ = 1$, as it should be.

Let us discuss the situation when $\kappa_1 \gg 1 \gg \kappa_0$, i.e. the creation of $\nu\bar{\nu}$ pairs is mainly owing to the noninertial effects. We can define the number of created pairs per second in the energy interval dE as $d\dot{N} = P_- dE/2\pi$. On the basis of eqs. (4.18) and (5.11), we get the distribution of created particles in this limit as

$$\frac{d\dot{N}}{dE} = \exp\left(-\frac{E + \delta E}{T_{\text{eff}}}\right), \quad T_{\text{eff}} = \frac{a}{2\pi}, \quad \delta E = \frac{p_{\perp}^2 + m^2}{8|V|}. \quad (5.12)$$

Here we reproduce the temperature of the Unruh radiation T_{eff} [8]. The correction to the Unruh effect owing to the specific electroweak neutrino radiation is given by the quantity δE in eq. (5.12). Note that eq. (5.12) is obtained under the assumption that $E \gg \delta E$.

6 Neutrino pairs creation in SN

In this section we shall discuss a possible application of the obtained results for the description of the $\nu\bar{\nu}$ pairs creation in a core collapsing SN. In particular, we shall obtain the astrophysical upper limit on the neutrino mass.

The distribution of created $\nu\bar{\nu}$ pairs is given in eq. (5.12). If we discuss the case of the neutrinos propagation along the z -axis, the correction to the Unruh effect does not suppress the pair creation if $\delta E \ll T_{\text{eff}}$. This condition can be rewritten as the constraint on the neutrino mass: $m \ll m_{\text{cr}}$, where

$$m_{\text{cr}} = 2\sqrt{\frac{|V|a}{\pi}}. \quad (6.1)$$

Note that the constraint in eq. (6.1) is theoretical. It means that, if $\nu\bar{\nu}$ pairs, created because of the vacuum instability in an accelerated matter, are observed experimentally, then the neutrino mass should be less than m_{cr} in eq. (6.1).

Let us evaluate m_{cr} for a core collapsing SN. When the plasma pressure cannot equilibrate the gravity of the upper layers of a protostar, the radius of a protostar core starts to decrease until the matter density reaches $\rho_n \sim 10^{14} \text{ g} \cdot \text{cm}^{-3}$. Assuming that PNS mainly consists of neutrons, this value corresponds to the neutron density $n_n \approx 6 \times 10^{37} \text{ cm}^{-3}$. At this point no further core compression happens and initially falling matter bounces from a dense core which then turns out to be PNS. According to the results of ref. [13], the matter velocity changes by $|\Delta v| \approx 5 \times 10^9 \text{ cm} \cdot \text{s}^{-1}$ during $\Delta t \approx 1 \text{ ms}$ after a bounce. It gives the nonzero matter acceleration $a = |\Delta v|/\Delta t \approx 5 \times 10^{12} \text{ cm} \cdot \text{s}^{-2}$. Using eq. (2.4) and the chosen values of all the parameters, we get that $m_{\text{cr}} \approx 7.2 \times 10^{-7} \text{ eV}$. The obtained value of m_{cr} is comparable with the (theoretical) upper bounds on the neutrino masses derived in ref. [28].

The total number of $\nu\bar{\nu}$ pairs emitted during Δt from the whole surface of PNS can be obtained by integrating eq. (5.12) over E and \mathbf{p}_{\perp} . The final result reads

$$N = 8|V|T_{\text{eff}}^2 \Delta t R^2 \exp\left(-\frac{m^2}{8|V|T_{\text{eff}}}\right), \quad (6.2)$$

where R is the PNS radius. The maximal matter acceleration at a bounce happens at $R \sim 10^3 \text{ km}$ [13]. Therefore, assuming that $m \sim m_{\text{cr}}$, on the basis of eq. (6.2) one gets that $N \sim 10^{11}$ neutrino pairs can be emitted by PNS.

This $\nu\bar{\nu}$ flux is much less than the flux of ν_e emitted in a neutrino burst, which can be estimated as $N_{\nu_e} \sim 10^{54}$ [29]. However, the typical energy of ν and $\bar{\nu}$, created owing to the vacuum instability in a dense nuclear matter in PNS, is in the eV range; cf. ref. [28], and $E_{\nu_e} \sim 10$ MeV. Therefore, at the bounce stage, high energy ν_e 's will be absorbed by the dense matter of PNS whereas low energy ν and $\bar{\nu}$ will freely escape PNS. Indeed, using the neutrino scattering cross sections given in ref. [30], one finds that the mean free path of ν and $\bar{\nu}$, with energies in eV range, in background matter with the density $\rho_n \sim 10^{14} \text{ g}\cdot\text{cm}^{-3}$ is about 10^{10} km , which is far beyond $R = 10^3 \text{ km}$. Thus one can consider the emission of $\nu\bar{\nu}$ pairs created by the accelerated matter in PNS as a precursor of the neutrino burst.

Nevertheless, it is unlikely that such ν and $\bar{\nu}$, created by the proposed mechanism, can be detected using modern experimental techniques. Despite the recent suggestions to observe nonrelativistic relic neutrinos with sub-eV energies [31], the flux of $\nu\bar{\nu}$ pairs emitted in a SN explosion is quite low to be detected in any existing neutrino telescope even if this explosion happens in our Galaxy.

7 Conclusion

In conclusion we mention that, in the present work, we have studied the evolution of neutrinos electroweakly interacting with a linearly accelerated background matter. We have briefly reminded the standard model neutrino interaction with background fermions in the flat space-time in section 2. Then, in section 3, we have derived the Dirac equation for a massive neutrino, electroweakly interacting with background fermions, in the Rindler space-time corresponding to a linearly accelerated matter. The solution of this Dirac equation has been found for ultrarelativistic particles. The quantum states of neutrinos have been analyzed in section 4. In section 5, we have applied our results for the description of the $\nu\bar{\nu}$ pairs creation in an accelerated matter. Finally, in section 6, we have considered the $\nu\bar{\nu}$ pairs creation in a collapsing SN at the bounce stage.

As a main tool in our study we have used the method of exact solutions of the Dirac equation for a massive neutrino rewritten in a comoving frame, where matter is at rest. Firstly, this method enables one to unambiguously determine the neutrino interaction potentials with background fermions which move with an acceleration [6, 12]. Secondly, unlike refs. [4, 5], where the matter acceleration was accounted for only in the neutrino refraction index, our approach allowed us to examine noninertial effects.

We have demonstrated that the neutrino interaction with a linearly accelerated matter can induce the vacuum instability which leads to the creation of $\nu\bar{\nu}$ pairs. We have discussed the situation when an observer was in the accelerated frame since we used the coordinates in the Rindler wedge. Therefore, the main contribution to the $\nu\bar{\nu}$ pairs creation is owing to the Unruh effect [8]. We have rederived the temperature of the Unruh thermal radiation and studied the correction to the Unruh effect due to the specific electroweak neutrino interaction.

The electroweak decays of accelerated particles were studied refs. [32, 33]. The probability of the decay of an accelerated proton $p \xrightarrow{a} e^+ n \nu_e$ was shown in refs. [32, 33] to be nonzero. In those works the hadronic current was described quasiclassically in the flat

space-time. Moreover it was demonstrated that the decay rate is equal to the sum of the decay rates of cross symmetric processes. These cross symmetric processes were studied using the exact solutions of the Dirac equation in the Rindler wedge. The results of refs. [32, 33] may be interpreted as the indirect proof of the existence of the Unruh effect.

Unlike refs. [32, 33], in the present work we consider the elastic forward neutrino scattering off accelerated background fermions f with a possible creation of $\nu\bar{\nu}$ pairs: $\nu f \xrightarrow{a} \nu f(\nu\bar{\nu})$. We treat background fermions as a classical external current. Therefore we confirm the result of refs. [32, 33] that the Unruh effect for neutrinos takes place if background fermions are treated quasiclassically.

In our analysis we use the exact solution of the Dirac equation which accounts for both the interaction with matter and its acceleration, i.e. our results exactly take into account all terms in the perturbative expansion over G_F . It is the main difference from refs. [32, 33], where the electroweak decays were studied on the tree level linear in G_F . The chosen way to describe neutrinos interacting with an accelerated matter allowed us not only establish the existence of the Unruh effect for these particles but also to obtain the correction to this effect in eq. (5.12).

The obtained electroweak correction to the Unruh effect was shown to contribute to the $\nu\bar{\nu}$ pairs creation in a core collapsing SN. Assuming that the process of the pairs creation is not suppressed, we have obtained the upper (theoretical) limit to the neutrino mass; cf. eq. (6.1). This constraint is in agreement with the result of ref. [28]. Unfortunately, nowadays it is almost impossible to experimentally detect ν and $\bar{\nu}$ created in the proposed mechanism.

It should be noted that previously the creation of $\nu\bar{\nu}$ pairs in dense matter of a neutron star (NS) was considered in refs. [34–36] using the Schwinger mechanism. For the first time the study of this problem based of the quantum field theory was undertaken in ref. [28]. However, in ref. [28] the case of the time dependent effective potential of the neutrino interaction with the background matter was discussed. Thus the model considered in ref. [28] corresponds to the neutronization stage of PNS.

The process of the $\nu\bar{\nu}$ pairs creation described in the present work can be attributed to the dependence of the external field (background matter) on spatial coordinates. The general description of the vacuum instability in case of coordinate dependent external fields was recently discussed in refs. [26, 27]. In our work for the first time we consider the coordinate dependent effective potential of the neutrino interaction with background matter as a source of the neutrino vacuum instability.

Finally we mention that the creation of $\nu\bar{\nu}$ pairs considered in the present paper is different from the bremsstrahlung emission of $\nu\bar{\nu}$ pairs in the NS matter; cf. refs. [29, 37]. Neutrino pairs emitted in the latter effect have energies proportional to the temperature of the nuclear matter in NS and thus correspond to high energies. On the contrary, we predict the coherent emission of low energy particles.

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A Properties of Whittaker functions

In this appendix we list some of the useful properties of Whittaker and Euler gamma functions. These expressions are taken from ref. [38].

The asymptotic expansions of Whittaker functions read

$$M_{\lambda,\mu}(z) \sim z^{1/2+\mu} + \dots, \quad \text{at } |z| \rightarrow 0, \quad (\text{A.1})$$

$$W_{\lambda,\mu}(z) \sim z^\lambda e^{-z/2} + \dots, \quad \text{at } |z| \rightarrow \infty. \quad (\text{A.2})$$

The derivatives of the Whittaker functions have the form,

$$\begin{aligned} \frac{dW_{\pm i\kappa,\mu}(\pm i\rho)}{d\rho} &= \frac{e^{\pm 3i\pi/4}}{\sqrt{\rho}} \left[\kappa \mp i \left(\mu - \frac{1}{2} \right) \right] W_{\pm i\kappa-1/2,\mu-1/2}(\pm i\rho) \\ &\quad - \left(\frac{2\mu-1}{2\rho} \pm \frac{i}{2} \right) W_{\pm i\kappa,\mu}(\pm i\rho), \\ \frac{dM_{\pm i\kappa,\mu}(\pm i\rho)}{d\rho} &= \frac{2\mu e^{\pm i\pi/4}}{\sqrt{\rho}} M_{\pm i\kappa-1/2,\mu-1/2}(\pm i\rho) - \left(\frac{2\mu-1}{2\rho} \pm \frac{i}{2} \right) M_{\pm i\kappa,\mu}(\pm i\rho), \\ \frac{dW_{\pm i\kappa,\mu}(\pm i\rho)}{d\rho} &= - \frac{e^{\pm i\pi/4}}{\sqrt{\rho}} W_{\pm i\kappa+1/2,\mu-1/2}(\pm i\rho) - \left(\frac{2\mu-1}{2\rho} \mp \frac{i}{2} \right) W_{\pm i\kappa,\mu}(\pm i\rho), \\ \frac{dM_{\pm i\kappa,\mu}(\pm i\rho)}{d\rho} &= \frac{2\mu e^{\pm i\pi/4}}{\sqrt{\rho}} M_{\pm i\kappa+1/2,\mu-1/2}(\pm i\rho) - \left(\frac{2\mu-1}{2\rho} \mp \frac{i}{2} \right) M_{\pm i\kappa,\mu}(\pm i\rho). \end{aligned} \quad (\text{A.3})$$

At $\mu = \lambda \pm 1/2$, $W_{\lambda,\mu}(z)$ takes the form,

$$W_{\lambda,\lambda-1/2}(z) = z^\lambda e^{-z/2}, \quad W_{\lambda,\lambda+1/2}(z) = z^{-\lambda} e^{z/2} \Gamma(1+2\lambda, z), \quad (\text{A.4})$$

where $\Gamma(a, z)$ is the incomplete gamma function.

The function $M_{\lambda,\mu}(z)$ is related to $W_{\lambda,\mu}(z)$ by the following expression:

$$M_{\lambda,\mu}(z) = \Gamma(2\mu+1) e^{-i\pi\lambda} \left[\frac{W_{-\lambda,\mu}(e^{-i\pi}z)}{\Gamma(1/2+\mu-\lambda)} + e^{i\pi(\mu+1/2)} \frac{W_{\lambda,\mu}(z)}{\Gamma(1/2+\mu+\lambda)} \right], \quad (\text{A.5})$$

which is valid if $-\pi/2 < \arg z < 3\pi/2$ and $2\mu \neq -1, -2, \dots$.

The absolute value of the Euler gamma function of the complex argument in some cases can be evaluated as

$$\left| \Gamma\left(\frac{1}{2} + ix\right) \right|^2 = \frac{\pi}{\cosh \pi x}, \quad |\Gamma(1+ix)|^2 = \frac{\pi x}{\sinh \pi x}, \quad (\text{A.6})$$

where x is a real parameter.

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